



امتحان الفصل
للعام الجامعي 2025/2024

المادة: حساب التفاضل والتكامل

المدة:

الدورة: الأولى

المرحلة:

السنة المنهجية: الثانية

الاستاذ: عباس رمال

Problem 1 (8 points)

Consider the function g defined by:

$$g(x) = \begin{cases} \ln(1 + 2x), & x \geq 0, \\ e^{-x} - \sqrt{1 + x^2}, & x < 0. \end{cases}$$

1. Show that g is continuous at $x=0$.
2. Study the differentiability of g at $x=0$.
3. Determine if the **Mean Value Theorem** can be applied to g over $[-1,1]$.
4. If the theorem applies, find a point $c \in (-1,1)$ such that:

$$g'(c) = \frac{g(1) - g(-1)}{2}.$$

Problem 2: (12 points)

Consider the function F defined by:

$$F(x) = \begin{cases} x^3 - 3x + 2, & x \leq 2, \\ \sqrt{x+2} - x, & x > 2. \end{cases}$$

$f(x) = (x-2)(x^3 - 3x + 2)$

1. Show that F is continuous at $x=2$.
2. Study the differentiability of F at $x=2$.
3. Verify if Rolle's Theorem can be applied to F on $[0,2]$. If applicable, find all points $c \in (0,2)$ such that $F'(c)=0$.

4. Determine if the Mean Value Theorem can be applied to F on $[1,3]$. If applicable, find a point $c \in (1,3)$ such that

$$F'(c) = \frac{F(3) - F(1)}{2}.$$

5. Compute the one-sided derivatives at $x=2$ using limits:

$$\lim_{x \rightarrow 2^-} \frac{F(x) - F(2)}{x - 2}, \quad \lim_{x \rightarrow 2^+} \frac{F(x) - F(2)}{x - 2}.$$

Comment on differentiability using these limits.
